

APPROFONDIMENTI

①

Determiniamo ora alcune importanti relazioni che legano le grandezze precedentemente introdotte.

Per prima cosa: $M = M_1 \cdot M_2$ (1)

$$\Delta_m = [\sqrt{M_1 M_2}] ; (2) ; \quad \beta = (1 - \alpha^2) ; (3)$$

$$M_2 = K M_1 ; (4) ;$$

$$\Delta_0 = \frac{(M_1 + M_2)}{2} = \frac{(M_1 + K M_1)}{2} = \frac{(K+1) \cdot M_1}{2}$$

$$\Delta_0 = \frac{(K+1)}{2} M_1 ; (5)$$

$$\frac{\Delta_0}{\Delta_m} = \frac{(K+1)}{2} \cdot \frac{M_1}{\Delta_m} ; (6)$$

$$\Delta_m = [\sqrt{M_1 M_2}] = [\sqrt{M_1 \cdot K M_1}] \approx \sqrt{K} \cdot M_1$$

$$\Delta_m = \sqrt{K} \cdot M_1 ; (7) \Rightarrow \Delta_0 = \frac{(K+1)}{2\sqrt{K}} \cdot \Delta_m$$

Si può anche scrivere:

$$\Delta_0 = \frac{(K+1) \cdot M_1}{2} \Rightarrow M_1 = \frac{2 \cdot \Delta_0}{(K+1)} = \frac{2 \cdot (K+1) \Delta_m}{2(K+1) \sqrt{K}} = \frac{\Delta_m}{\sqrt{K}}$$

$$M_1 = \frac{\Delta_m}{\sqrt{K}} (8)$$

$$\text{E ancora: } \Delta_0 = \frac{(M_1 + M_2)}{2} = \frac{(M_1 + \frac{M_2}{K})}{2} = \frac{(K M_1 + M_2)}{2K} =$$

$$= \frac{(k+1)}{2k} \cdot m_2$$

②

$$d_0 = \frac{(k+1)}{2k} m_2 \quad (9)$$

$$m_2 = \frac{2k}{(k+1)} \cdot d_0 = \frac{2k}{(k+1)} \cdot \frac{(k+1)}{2\sqrt{k}} \cdot d_m = \frac{k}{\sqrt{k}} \cdot d_m = \sqrt{k} \cdot d_m$$

$$m_2 \approx \sqrt{k} \cdot d_m \quad (10)$$

$$\begin{cases} m_1 = \frac{d_m}{\sqrt{k}} \\ m_2 \approx \sqrt{k} \cdot d_m \end{cases}; \quad (11)$$

$$g_m = (\sqrt{1} + \sqrt{2}) = 2 \cdot (d_0 - d_m) = 2 \cdot \left[\frac{(k+1)\sqrt{k}}{2k} d_m - d_m \right] =$$

$$= 2 d_m \left[\frac{k\sqrt{k} + \sqrt{k} - 2k}{2k} \right] = 2 d_m \left[\frac{(k+1)\sqrt{k} - 2\sqrt{k}\sqrt{k}}{2k} \right] =$$

$$= \frac{2 d_m \sqrt{k} (k+1 - 2\sqrt{k})}{2k} = d_m \frac{\sqrt{k}}{k} \cdot (\sqrt{k} - 1)^2$$

$$= \frac{(\sqrt{k} - 1)^2}{\sqrt{k}} \cdot d_m$$

$$g_m = \frac{(\sqrt{k} - 1)^2}{\sqrt{k}} \cdot d_m; \quad (12)$$

lim $g_m = 0$ se d_m e Lipschitz non crescente.
 $k \rightarrow 1$

Definiamo ora $M = \mu_1 \cdot \mu_2$ e $N = f_1 f_2 \mu_1 \mu_2$ (13) (3)

Indichiamo con $L_M = [\sqrt{\mu_1 \mu_2}]$; (14);

ed $L_N = [\sqrt{f_1 \cdot f_2 \cdot \mu_1 \mu_2}]$; (15);

Indichiamo con: $\frac{\mu_2}{\mu_1} = K_M$; (16); $\frac{f_2}{f_1} = K$; (17)

Supponiamo $K > K_M \Rightarrow \frac{f_2}{f_1} > \frac{\mu_2}{\mu_1}$; (18)

h'abbiamo: $K_N = \frac{K}{K_M}$; (19)

$K_N = \frac{f_2}{f_1} / \frac{\mu_2}{\mu_1} = \frac{f_2}{f_1} \cdot \frac{\mu_1}{\mu_2}$; (20)

$L_N = \sqrt{f_1 f_2 \cdot \mu_1 \mu_2} = \sqrt{f_1} \cdot \sqrt{f_2 \cdot \mu_1 \mu_2}$; (21)

ora dalle (11) si ha:

$$\begin{cases} \sigma_{1N} = \frac{(1 - \sqrt{K_N})}{\sqrt{K_N}} \cdot L_N \\ \sigma_{2N} = (\sqrt{K_N} - 1) \cdot L_N \end{cases} \quad (22)$$

$$h_{12} \frac{\delta_{1N}}{f_1} = \frac{(1 - \sqrt{K_N})}{f_1 \cdot \sqrt{K_N}} \cdot \sqrt{f_1} \cdot \sqrt{f_2} \cdot d_m = \quad (4)$$

$$= \frac{\left(1 - \sqrt{\frac{f_2}{f_1} \cdot \frac{\mu_1}{\mu_2}}\right)}{\sqrt{\frac{f_2}{f_1} \cdot \frac{\mu_1}{\mu_2}}} \cdot \sqrt{\frac{f_1 \cdot f_2}{f_1^2}} \cdot d_m =$$

$$= \frac{\left(1 - \sqrt{\frac{f_2}{f_1} \cdot \frac{\mu_1}{\mu_2}}\right)}{\sqrt{\frac{f_2}{f_1} \cdot \frac{\mu_1}{\mu_2}}} \cdot \sqrt{\frac{f_2}{f_1}} \cdot d_m =$$

$$= \left(\sqrt{\frac{\mu_2}{\mu_1}} - \sqrt{\frac{f_2}{f_1}}\right) \cdot d_m$$

$$h_1 = (\sqrt{K_N} - \sqrt{K}) \cdot d_m ; (23)$$

$$h_{2N} \frac{\delta_{2N}}{f_2} = \frac{(\sqrt{K_N} - 1)}{f_2} \cdot d_m = \frac{\left(\sqrt{\frac{f_2}{f_1} \cdot \frac{\mu_1}{\mu_2}} - 1\right)}{f_2} \cdot \sqrt{f_1} \cdot \sqrt{f_2} \cdot d_m =$$

$$= \frac{\left(\sqrt{\frac{f_2}{f_1} \cdot \frac{\mu_1}{\mu_2}} - 1\right)}{1} \cdot \sqrt{\frac{f_1 \cdot f_2}{f_2^2}} \cdot d_m =$$

$$= \left(\sqrt{\frac{f_2}{f_1} \cdot \frac{\mu_1}{\mu_2}} - 1\right) \sqrt{\frac{f_1}{f_2}} \cdot d_m = \left(\sqrt{\frac{\mu_1}{\mu_2}} - \sqrt{\frac{f_1}{f_2}}\right) \cdot d_m =$$

(5)

$$= \left(\sqrt{\frac{1}{K_m}} - \sqrt{\frac{1}{K}} \right) d_m = \frac{(\bar{V}_K - \bar{V}_{K_m})}{\bar{V}_K \bar{V}_{K_m}} \cdot d_m$$

$$\frac{1}{h_{22}} \frac{(\bar{V}_K - \bar{V}_{K_m})}{\bar{V}_K \cdot \bar{V}_{K_m}} \cdot d_m \quad (24)$$

$$y_2 = (f_1 + h_2) = (\bar{V}_{K_m} - \bar{V}_K) \cdot d_m + \frac{(\bar{V}_K - \bar{V}_{K_m})}{\bar{V}_K \bar{V}_{K_m}} \cdot d_m =$$

$$= \frac{(\bar{V}_{K_m} \bar{V}_{K_m} \bar{V}_K - \bar{V}_{K_m} \bar{V}_K \bar{V}_K + \bar{V}_K - \bar{V}_{K_m})}{\bar{V}_{K_m} \cdot \bar{V}_K} \cdot d_m =$$

$$= \frac{(K_m \bar{V}_K - K \bar{V}_{K_m} + \bar{V}_K - \bar{V}_{K_m})}{\bar{V}_{K_m} \cdot \bar{V}_K} \cdot d_m =$$

$$= \left(\frac{K_m}{\bar{V}_{K_m}} - \frac{K}{\bar{V}_K} + \frac{1}{\bar{V}_{K_m}} - \frac{1}{\bar{V}_K} \right) \cdot d_m =$$

$$= -(\bar{V}_K - \bar{V}_{K_m}) / d_m + \frac{(\bar{V}_K - \bar{V}_{K_m})}{\bar{V}_K \cdot \bar{V}_{K_m}} \cdot d_m =$$

$$= (\sqrt{k} - \sqrt{k_m}) \left(\frac{1}{\sqrt{k} \cdot \sqrt{k_m}} - 1 \right) \cdot \Delta \mu ;$$

⑥

$$g_2 = (\sqrt{k} - \sqrt{k_m}) \left(\frac{1}{\sqrt{k} \cdot \sqrt{k_m}} - 1 \right) \cdot \Delta \mu ; \quad (25)$$

$$k > k_m \Rightarrow g_2 < 0 ; \quad (26) ;$$

g_2 può essere posta anche nella forma:

$$g_2 = \frac{(\sqrt{k} - \sqrt{k_m}) \cdot (\sqrt{k} + \sqrt{k_m}) \cdot (1 - \sqrt{k} \cdot \sqrt{k_m})}{(\sqrt{k} + \sqrt{k_m}) \sqrt{k} \sqrt{k_m}} \cdot \Delta \mu \Rightarrow$$

$$g_2 = \frac{(k - k_m) (1 - \sqrt{k} \cdot \sqrt{k_m})}{(\sqrt{k} + \sqrt{k_m}) (\sqrt{k} \cdot \sqrt{k_m})} \Delta \mu ; \quad (27)$$

(7)

Consideriamo ora il caso ipocritico.

$$K < K_m \quad (28), \quad \text{con } K = \frac{f_2}{f_1} \quad \text{e} \quad K_m = \frac{m_2}{m_1};$$

$$\frac{f_2}{f_1} < \frac{m_2}{m_1}, \quad (29)$$

$$K_N = \frac{K_m}{K}, \quad (30)$$

$$K_m = \frac{m_2}{m_1} \cdot \frac{f_1}{f_2}, \quad d_N = \sqrt{f_1 f_2} \cdot d_m = \sqrt{f_1} \cdot \sqrt{f_2} \cdot d_m$$

$$\delta_{1N} = \frac{(1 - \sqrt{K_N})}{\sqrt{K_N}} \cdot d_N; \quad \delta_{2N} = (\sqrt{K_N} - 1) \cdot d_N$$

$$h_1 = \frac{\delta_{1N}}{f_2} = \frac{(1 - \sqrt{K_N})}{f_2 \sqrt{K_N}} \cdot d_N; \quad \Rightarrow$$

$$h_2 = \frac{\delta_{2N}}{f_1} = \frac{(\sqrt{K_N} - 1)}{f_1} \cdot d_N$$

$$h_1 = \frac{(1 - \sqrt{K_N})}{f_2 \cdot \sqrt{K_N}} \cdot \sqrt{f_1} \cdot \sqrt{f_2} \cdot d_m =$$

$$= \frac{\left(1 - \sqrt{\frac{m_2}{m_1} \cdot \frac{f_1}{f_2}}\right)}{\sqrt{\frac{m_2}{m_1} \cdot \frac{f_1}{f_2}}} \cdot \sqrt{\frac{f_1 f_2}{f_2^2}} \cdot d_m =$$

(8)

$$= \frac{\left(1 - \sqrt{\frac{\mu_2}{\mu_1} \cdot \frac{f_1}{f_2}}\right)}{\sqrt{\frac{\mu_2}{\mu_1}} \cdot \sqrt{\frac{f_1}{f_2}}} \cdot \cancel{\sqrt{\frac{f_1}{f_2}}} \cdot d_m =$$

$$= \left(\sqrt{\frac{\mu_1}{\mu_2}} - \sqrt{\frac{f_1}{f_2}}\right) d_m = \left(\frac{1}{\sqrt{K_m}} - \frac{1}{\sqrt{K}}\right) d_m$$

$$h_2 = \frac{(\sqrt{K} - \sqrt{K_m})}{\sqrt{K} \cdot \sqrt{K_m}} \cdot d_m \quad (31)$$

$$h_2 = \frac{(\sqrt{K_m} - 1)}{f_1} d_m = \frac{\left(\sqrt{\frac{\mu_2}{\mu_1} \cdot \frac{f_1}{f_2}} - 1\right)}{f_1} \sqrt{f_1} \cdot \sqrt{f_2} \cdot d_m =$$

$$= \left(\sqrt{\frac{\mu_2}{\mu_1}} \cdot \sqrt{\frac{f_1}{f_2}} - 1\right) \sqrt{\frac{f_2}{f_1}} \cdot d_m =$$

$$= \left(\sqrt{\frac{\mu_2}{\mu_1}} - \sqrt{\frac{f_2}{f_1}}\right) d_m = (\sqrt{K_m} - \sqrt{K}) \cdot d_m$$

$$h_2 = (\sqrt{K_m} - \sqrt{K}) d_m; \quad (32);$$

(9)

$$g_2 = (h_1 + h_2) = \frac{(\sqrt{k} - \sqrt{k_m})}{\sqrt{k} \cdot \sqrt{k_m}} \cdot d_m + (\sqrt{k_m} - \sqrt{k}) \cdot d_m =$$

$$= (\sqrt{k} - \sqrt{k_m}) \left(\frac{1}{\sqrt{k} \cdot \sqrt{k_m}} - 1 \right) \cdot d_m$$

$$g_2 = (\sqrt{k} - \sqrt{k_m}) \cdot \left(\frac{1}{\sqrt{k} \cdot \sqrt{k_m}} - 1 \right) \cdot d_m; \quad (33)$$

$$k < k_m \Rightarrow g_2 > 0; \quad (34):$$

Si osserva anche che:

$$k_m = \frac{m_2}{m_1} = \frac{N}{m_1^2} \Rightarrow \sqrt{k_m} = \frac{\sqrt{N}}{m_1} \simeq \frac{d_m}{m_1}$$

$$k_m \simeq \frac{d_m}{m_1}; \quad (35)$$

$$h_1 \cdot h_2 = \frac{(\sqrt{k} - \sqrt{k_m})}{\sqrt{k} \cdot \sqrt{k_m}} \cdot d_m \cdot (\sqrt{k_m} - \sqrt{k}) \cdot d_m =$$

$$= \frac{(\sqrt{k} \sqrt{k_m} - k - k_m + \sqrt{k} \sqrt{k_m})}{\sqrt{k} \cdot \sqrt{k_m}} \cdot d_m^2 = \frac{(2 \sqrt{k} \cdot \sqrt{k_m} - k - k_m)}{\sqrt{k} \cdot \sqrt{k_m}} \cdot d_m^2 =$$

$$= \left(2 - \frac{k + k_m}{\sqrt{k} \cdot \sqrt{k_m}} \right) d_m^2$$

$$h_1 \cdot h_2 = \left(2 - \frac{(k + k_m)}{\sqrt{k} \cdot \sqrt{k_m}} \right) d_m^2; \quad (36)$$

Esempio n°1

(10)

$$M_1 = 4999; M_2 = 6007; n = 4999 \times 6007 = 30\,028\,993;$$

$$\alpha_0 = 5480; \alpha_1 = 5503; \beta_0 = 46; \beta_1 = -1407;$$

$$x = \frac{y}{2} \pm \sqrt{\frac{y^2}{4} + 5480y + 1407} ;$$

$$y = 46 \Rightarrow x = 23 \pm 504 = \begin{pmatrix} -421 \\ 527 \end{pmatrix} \quad \left\{ \begin{array}{l} M_1 = (5480 - 421) = 4999 \\ M_2 = 5480 + 527 = 6007 \end{array} \right.$$

$$N = 10 \times 11 \times 4999 \times 6007 = 3.323\,189\,230$$

$$\alpha_0 = 10 \times 11 \times 523\,54530; \alpha_1 = 523; \beta = -6.511.670$$

$$\beta_0 = -59194; \beta_1 = -1;$$

$$x = \frac{y}{2} \pm \sqrt{\frac{y^2}{4} + 57530y + 6.511.670}$$

$$y = -1 \Rightarrow x = -\frac{1}{2} \pm \frac{5881}{2} = \begin{pmatrix} 2540 = 10 \times 254 \\ 2541 = -11 \times 231 \end{pmatrix} ;$$

$$y_2 = 254 - 231 = 23;$$

(91)

$$h = \frac{[y_2 + (p_2 - h_1/d_2)] \pm \sqrt{[y_2 + (p_2 - h_1/d_2)]^2 - 4(p_2 - p_1/d_2)y_2}}{2}$$

$$h = \frac{(23 + 523) \pm \sqrt{(23 + 523)^2 - 4(-59197 - 10523 \times 23)}}{2}$$

$$= \frac{546 \pm 1008}{2} = \begin{matrix} -231 \\ 777 \end{matrix}$$

$$h_1 = -231;$$

$$h_2 = 777 - h_1 = 777 - 523 = 254;$$

Ci poniamo ora il problema:

(19)

Dato un intero m_1 , quale è il valore massimo che può assumere m_2 , affinché nella relazione:

$$x = \frac{y}{2} \pm \sqrt{\frac{y^2}{4} + dy - \beta} \quad ; \quad (37)$$

si abbia $y=0$, ovvero: $\beta = z^2$; (38)

Bisogna precisare che se m_1 ed m_2 sono entrambi dispari, allora y può essere zero, mentre se m_1 ed m_2 sono uno pari ed uno dispari, allora y non potrà essere zero, ma uno.

Dato m_1 intero, poniamo:

$$m_2 = m_1 + 2t \quad \text{con } t \geq 0 \quad (39)$$

Dalla (38):

$$t^2 < m_1 + m_1 + 2t; \quad (40)$$

$$t^2 - 2t - 2m_1 < 0 \Rightarrow 0 < t < \left[1 + \sqrt{1 + 2m_1} \right]$$

$$K = \frac{m_1 + 2 \cdot \left[1 + \sqrt{1 + 2m_1} \right]}{m_1}; \quad (41)$$

Ad exemplo per $\mu_1 = 10001$, n' altra:

(13)

$$\mu_2 = 10285$$

$$\text{com } 2t = 284,$$

per $\mu_1 = 90001$ n' altra:

$$\mu_2 = 80803; \text{ con } t = 401;$$

per $\mu_1 = 99103$ n' altra:

$$\mu_2 = 89909 \text{ con } t = 403;$$